Universal *CP*-Nonconserving Theories*

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The assumptions of the normal phenomenological weak-interaction theory are examined, and successively relaxed with a view to listing systematically all theories which are in varying degrees similar to the usual theory. A theory is found, of the current-current form, which is universal in the conventional sense, uses only $V-A$ currents, is capable of yielding CP violation, and is consistent with all existing experimental results.

THE discovery of CP violation in the decay of neutral K mesons[1] has compelled us to reexamine the basic structure of our theories of the elementary particles, since, as presently constructed, they are incapable of describing the experimental situation. In principle, modifications to our present ideas could appear in the strong interactions, in the electromagnetic interactions, or in the weak interactions, or an entirely new interaction not previously noticed might exist which incorporates CP violation. In fact, theories of all of these types have been proposed.

It is our intention here to assume that the modifications will appear as part of the theory of weak interactions, and, adopting the most conservative approach possible, to ask, on the basis of the few general principles that we trust, what forms this modification could take. To this end, let us begin by listing, in decreasing order of generality, the assumptions which describe our present theory of weak interactions.

The first assumption made in constructing the phenomenological weak interaction is that the Hamiltonian is of the current-current form; that is, one assumes the existence of a weak current j and writes

$$\mathcal{H}_{\rm wk}=g(jj^\dagger+j^\dagger j)\,, \tag{1}$$

where g is some coupling constant.

In the conventional theory, j is a Lorentz four-vector and carries charge $+1$. Presumably, however, other types of currents, with different Lorentz properties and/or different charges could exist as well. In this event the current-current hypothesis would, one expects, take the form

$$\mathcal{H}_{\rm wk}=\sum_i g_i(j_ij_i^\dagger+j_i^\dagger j_i)\,. \tag{2}$$

The second basic assumption is that of universality. This is usually stated by requiring that the current j is a sum of various pieces (a lepton and a hadron piece in the conventional theory), each of which commutes like the $\sigma_x+i\sigma_y$ component of an $SU(2)$ algebra: Specifically, one writes $j=\sum_\alpha j^\alpha$ and assumes that

$$[j^\alpha[j^\alpha,j^{\alpha\dagger}]]=-2j^\alpha. \tag{3}$$

If the Hamiltonian were of the more general form written in Eq. (2), one would expect the universality statement to apply to each of the currents j_i. Thus,

$$[j_i^\alpha[j_i^\alpha,j_i^{\alpha\dagger}]]=-2j_i^\alpha. \tag{4}$$

The coupling constants g_i for each current are then chosen to be equal: $g_i\equiv\sqrt{2}G$; this constitutes universality.

The third assumption is that the current is the sum (or difference) of a vector and an axial-vector piece. Explicitly, in the usual theory,

$$j^\alpha=V_\mu{}^\alpha-A_\mu{}^\alpha. \tag{5}$$

The absolute strength of the V and A currents can again be defined through the current-commutation relations. If several different currents j_i are allowed, this assumption would be stated

$$j_i{}^\alpha=V_{i\mu}{}^\alpha-A_{i\mu}{}^\alpha. \tag{6}$$

The final assumption of the conventional weak-interaction theory, as we mentioned earlier, is that there is a single current and that it carries charge $+1$.

We wish to ask what the most general possible weak interaction theory compatible with each stage of this list of assumptions is. We shall restrict ourselves to currents obtainable in the quark model, that is, to scalar (S), pseudoscalar (P), vector (V), axial-vector (A), and tensor (T) currents.

With this restriction, we can write for $\mathcal{H}_{\rm wk}$ the expression

$$\begin{aligned}\mathcal{H}_{\rm wk}=&\sum_i g_i{}^0(\alpha_i{}^0S_i+\beta_i{}^0P_i)(\alpha_i{}^{0*}S_i{}^\dagger+\beta_i{}^{0*}P_i{}^\dagger)\\&+\sum_i g_i{}^1(\alpha_i{}^1V_{i\mu}+\beta_i{}^1A_{i\mu})(\alpha_i{}^{1*}V_{i\mu}{}^\dagger+\beta_i{}^{1*}A_{i\mu}{}^\dagger)\\&+\sum_i g_i{}^2(\alpha_i{}^2T_{i\mu\nu}+\beta_i{}^2\bar{T}_{i\mu\nu})(\alpha_i{}^{2*}T_{i\mu\nu}{}^\dagger+\beta_i{}^{2*}\bar{T}_{i\mu\nu}{}^\dagger)\\&+(\text{symmetrized})\,,\end{aligned} \tag{7}$$

where $\bar{T}_{\mu\nu}\equiv\epsilon_{\mu\nu\lambda\sigma}T_{\lambda\sigma}$ is the dual tensor to T. The index i in the above equation now refers to whatever internal quantum numbers we wish to attach to the assorted

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

currents, and to distinguish between lepton and hadron currents. The theory at this stage evidently has sufficient parameters available to it, and more so, to be compatible with any known experimental result.[2]

Let us next impose the second assumption. First, notice that for the leptons, universality is conventionally expressed by considering $(e\nu_e)$ and $(\mu\nu_\mu)$ to form spin-$\frac{1}{2}$ basis vectors of an $SU(2)$. The requirement of universality is therefore equivalent to saying that for leptons only charged currents can come into play. For hadrons, on the other hand, if we assume the quark model, the possible currents are components of an $SU(3)$ octet. Hence, two independent $SU(2)$'s exist, one of which may be composed entirely of neutral currents. Thus, for hadrons, universality allows the possibility of both charged and neutral currents. To be specific, then, we have

(i) charged S, P, and/or V, A; and/or T, $\bar{T}$, lepton currents, and

(ii) charged and/or neutral S, P; and/or V, A; and/or T, $\bar{T}$ hadron currents.

Furthermore, universality imposes very severe restrictions on the coefficients α and β appearing in the general form written in Eq. (7).

If we now proceed to impose the third assumption, that of $V-A$ currents, the possibilities become very limited indeed. The available currents are limited to a charged $V-A$ lepton current, and charged and/or neutral $V-A$ hadron currents. The form of the Hamiltonian must be

$$\mathcal{H}_{\mathrm{wk}}=\sqrt{2}G[(j_{l\mu}+j_{h\mu}{}^{Q=1})(j_{l\mu}{}^{\dagger}+j_{h\mu}{}^{Q=1\dagger}) +j_{h\mu}{}^{Q=0}j_{h\mu}{}^{Q=0\dagger}]+(\text{symmetrized}), \quad (8)$$

where each j is of the form $V-A$, the subscripts l and h refer to leptons and hadrons, and the superscript $Q=0, 1$ refers to the charge carried by the hadron current.

The fourth and final assumption which we listed above, to the effect that there is only one $V-A$ current and that it is charged, leads us to the conventional theory,[3] which excludes CP violation and which cannot, therefore, explain the experimental observations.

The most conservative point of view is to try to retain as many of the conventional assumptions as possible; let us therefore begin by relaxing the assumption of least generality, namely, that of a single-charged $V-A$ current. We find ourselves, then, at the theory given in Eq. (8).

In this theory, the lepton current is the usual one. The $Q=1$ hadron current is forced by universality to be nearly the usual one, differing from it only in the existence of a relative phase between the strangeness-zero and strangeness-one parts of the current. This phase would be unmeasurable but for the existance of the neutral hadron current, and is in any case irrelevant except in nonleptonic processes. So far as the leptonic and semileptonic weak interactions are concerned, then, the conventional theory is unaltered, and we may henceforth confine ourselves solely to the nonleptonic part of $\mathcal{H}_{\mathrm{wk}}$.

The $SU(3)$ structure of the nonleptonic Hamiltonian is dictated by universality. We find, by imposing the commutation relations in Eq. (4), that the most general possible Hamiltonian has the form

$$\mathcal{H}_{\mathrm{wk}}=\sum_{\Delta S=0,\pm1,\pm2}\mathcal{H}_{\Delta S}, \quad (9)$$

where

$$\mathcal{H}_{\Delta S=0}=\sqrt{2}G[\cos^2\theta_C\,\{i_+,i_-\}+\sin^2\theta_C\,\{v_+,v_-\} +\tfrac{1}{2}(1+\cos^2\theta_N)\{u_+,u_-\}+\sin^2\theta_N\{u_z,u_z\}], \quad (10)$$

$$\mathcal{H}_{\Delta S=1}=\mathcal{H}_{\Delta S=-1}{}^{\dagger}=\sqrt{2}G[\tfrac{1}{2}\sin 2\theta_C\,e^{i\phi_C}\{v_-,i_+\} +\tfrac{1}{2}\sin 2\theta_N\,e^{i\phi_N}\{u_-,u_z\}], \quad (11)$$

$$\mathcal{H}_{\Delta S=2}=\mathcal{H}_{\Delta S=-2}{}^{\dagger}=\sqrt{2}G[-\tfrac{1}{4}\sin^2\theta_N\,e^{2i\phi_N}\{u_-,u_-\}]. \quad (12)$$

The notation here is as follows. The symbols i, v, and u stand for $V-A$ currents which transform under $SU(3)$ like I, V, or U spin. The subscript indicates which component of I, V, or U spin is involved. Thus, for example, $i_+=i_-{}^{\dagger}=\frac{1}{2}[(V_{1\mu}-A_{1\mu})+i(V_{2\mu}-A_{2\mu})]$; and $u_z=\frac{1}{4}[-(V_{3\mu}-A_{3\mu})+\sqrt{3}(V_{8\mu}-A_{8\mu})]$. In particular, i_+ and v_+ are the two parts of the usual Cabibbo current.

The Hamiltonian given above leads to the following qualitative consequences: (i) The part of $\mathcal{H}_{\Delta S=1}$ coming from neutral currents transforms under $SU(3)$ like a pure **27**, while that coming from charged currents, of course, includes pieces transforming like **8**. (ii) The neutral current gives rise to $\Delta S=2$ terms. (iii) CP violation exists provided the relative phase $\Delta\phi=\phi_C-\phi_N$ between the charged and neutral currents is not zero.[4]

The theory given in Eqs. (9)–(12) contains four parameters, θ_C, θ_N, ϕ_C, and ϕ_N. Of these, one is unmeasurable, and one (θ_C) is the usual Cabibbo angle. There are, therefore, two new parameters to be determined by looking at the phenomenology of the neutral K system.

For the theory given by Eqs. (9)–(12), the allowable range of the two new parameters $\Delta\phi$ and θ_N can be determined from the known experimental information about the mass difference Δm and the parameter ϵ,[5]

[2] Except the possibility of CPT violation; the form given in Eq. (7) conserves CPT.

[3] Actually, universality leads us to the usual Cabibbo theory but with an arbitrary relative phase between the strangeness-zero and strangeness-one parts of the hadron currents. This phase appears in the same way as a phase in the K-meson state, and is unmeasurable.

[4] The experimental consequences of this Hamiltonian are very similar to those in a theory of CP violation proposed by R. J. Oakes [Phys. Rev. Letters **20**, 1539 (1968)], for nonleptonic processes. For leptonic decays, they are different. Oakes imposes a very different form of universality, which gives neutral lepton currents and a different kind of hadron current than we have here. We wish to thank Dr. Jacques Weyers for calling this theory to our attention.

[5] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 383 (1964). We define $\eta_{+-}=\epsilon+\epsilon'$, $\eta_{00}=\epsilon-2\epsilon'$.

and the parameter ϵ' can then be predicted. There are basically two cases, distinguished by $0 \lesssim \Delta\phi < \frac{1}{2}\pi$ and $\Delta\phi \sim \frac{1}{2}\pi$. The physical consequences in both situations are essentially the same; they differ only in that in the first the CP violation is primarily associated with the existence of $\mathcal{H}_{\Delta S=2}$ (this is a kind of superweak theory[6]) while in the second the *CP* violation comes predominantly from $\mathcal{H}_{\Delta S=1}$ (this is, in a sense, a maximally *CP*-violating theory).

The contribution of the $\Delta S=2$ term to the mass difference Δm can be estimated by relating it to the experimental $K^+ \to \mu^+\nu$ rate, in the approximation of retaining only the vacuum intermediate state in $\langle \bar{K}_0 | \mathcal{H}_{\Delta S=-2} | K_0 \rangle$.[7] From the requirement that the total $\Delta S=2$ contribution coming from both the $\Delta S=1$ and 2 terms not be larger than the experimental value of Δm (which we take as $\Delta m / m_K = 6.5 \times 10^{-15}$), we obtain the restriction

$$\sin^2\theta_N \cos 2\Delta\phi \lesssim 2.5 \times 10^{-7}. \tag{13}$$

Similarly, the size of the $\Delta S=2$ contribution to ϵ is constrained to be no bigger than the experimental value of $\mathrm{Re}\,\epsilon$, which we take to be about 10^{-3}. This yields

$$\sin^2\theta_N \sin 2\Delta\phi \lesssim 10^{-9}. \tag{14}$$

Equations (13) and (14) limit the value of θ_N to be small; we have

$$\theta_N^2 \lesssim 2.5 \times 10^{-7}. \tag{15}$$

Now, if $\Delta\phi$ is not close to $\frac{1}{2}\pi$, the $\Delta S=2$ contribution to ϵ is the dominant one, and the equality obtains in Eq. (14). A range of possibilities then exists, with θ_N varying from 5×10^{-4} (with $\Delta\phi \sim \frac{1}{3} \times 10^{-2}$) down to $\theta_N \sim 3 \times 10^{-5}$ (corresponding to $\sin 2\Delta\phi \sim 1$). If, on the other hand, $\Delta\phi$ is close to $\frac{1}{2}\pi$, the $\Delta S=2$ term is not the most significant one in ϵ, and θ_N takes on its maximum value of 5×10^{-4}, as determined by Eq. (15).

[6] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).

[7] This type of estimate is made by Oakes (Ref. 4); our result agrees with his.

An estimate of the parameter ϵ' is

$$\epsilon' \sim \lambda \sin\Delta\phi \; (\theta_N / \theta_C), \tag{16}$$

where λ is a parameter describing the effect of octet enhancement on K-meson decays. Empirically, $\lambda \sim 1/25$. In all cases, $\epsilon' \ll \epsilon$, so that we should expect $|\eta_{+-}| = |\eta_{00}|$.

With these estimates of our parameters, we may now turn to other consequences.

In strangeness-violating $\Delta S=1$ processes, all *CP*- or *T*-violating effects will be very small, of the order $\sin\theta_N \sin\Delta\phi \sim 5 \times 10^{-5}$. The effects may well be even smaller, since the part of $\mathcal{H}_{\Delta S=1}$ coming from the neutral currents transforms as a pure **27** under $SU(3)$, and its effects may therefore be suppressed relative to the conventional part due to octet enhancement. For strangeness-conserving processes, there is no *CP* violation at all as we see from Eq. (10). There is, therefore, no neutron electric dipole moment.[8]

Nevertheless, effects of the neutral currents do show up in $\mathcal{H}_{\Delta S=0}$, and with strength comparable to that of the conventional charged-current terms. (Universality, after all, does force the neutral-current effect to be large *somewhere*.) The principal effect that the neutral currents have on $\Delta S=0$ weak processes is that they allow $\Delta I=1$ transitions, while the usual charged currents give rise mainly to $\Delta I=0$ or 2, the $\Delta I=1$ terms being suppressed by $\sin^2\theta_C \sim 1/20$. A new prediction of the theory is, then, that $\Delta I=1$ weak processes in nuclei should be comparable to those with $\Delta I=0$ or 2.

Finally, let us conclude by remarking that if we wish to be still more adventurous, we can try to relax the next basic assumption of the weak interaction, namely, that of the $V-A$ form. This brings us to an interaction of the form of Eq. (7). We have studied the details of this more general and much more complicated situation, and will describe them elsewhere.

[8] The present experimental upper limit is $\lesssim 2 \times 10^{-22}$ cm; see P. D. Miller, W. B. Dress, J. K. Baird, and N. F. Ramsey, Phys. Rev. Letters **19**, 381 (1967); C. G. Shull and R. Mathews, *ibid.* **19**, 384 (1967).